

# COSINE LAW

## Learnings Goal

- find sides and angles in non-right angled triangles

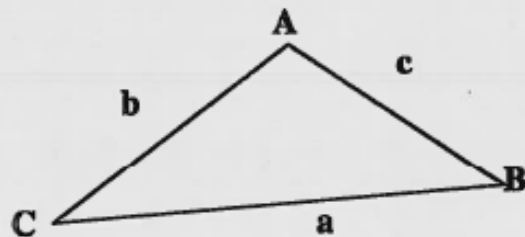
What happens if we don't know the measure of an angle and its opposite side? Can we solve such a non-right triangle?

The cosine law says that for any  $\triangle ABC$ ,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

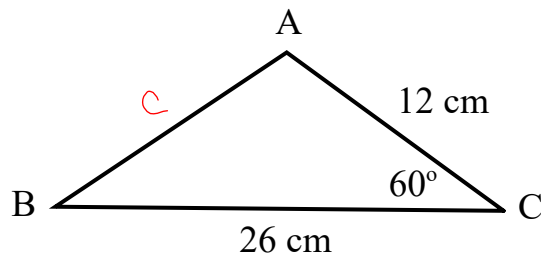


\* To use the cosine law you need to know at least 2 sides of a triangle.

\* If 2 sides of the triangle are known, and the contained angle, you can find the length of the third side.

\* If the length of the three sides of the triangle are known, you can find the unknown angle measurement.

Find side c.



Check which method you need?

Pythagorean Theorem

SohCahToa

Sine Law

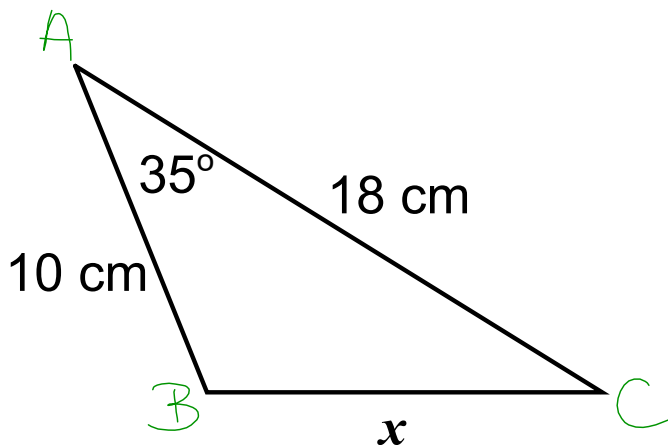
Cosine Law ✓

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 26^2 + 12^2 - 2(26)(12) \cos 60^\circ$$

$$\sqrt{c^2} = \sqrt{508}$$

$$c = 22.54$$



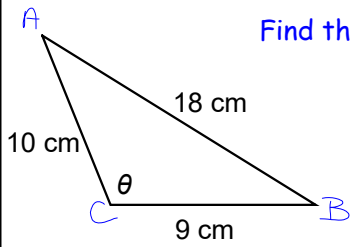
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 18^2 + 10^2 - 2(18)(10) \cos 35^\circ$$

$$\sqrt{a^2} = \sqrt{129.105}$$

$$a = 11.4$$

Find the angle  $\theta$  (theta)



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$18^2 = 9^2 + 10^2 - 2(9)(10) \cos C$$

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$$324 = 81 + 100 - 180 \cos C$$

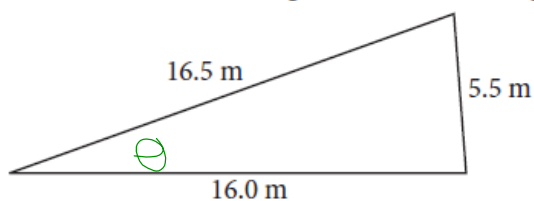
$$243 = 100 - 180 \cos C$$

$$143 = -180 \cos C$$

$$-0.79 = \cos C$$

$$142^\circ = C$$
**On the Boards...**

A motocross ramp is to be built for an upcoming race. The measures for the sides of the ramp are as shown. Calculate the angle of inclination of the ramp to the nearest degree.



Decide which method to use.

$$5.5^2 = 16^2 + 16.5^2 - 2(16)(16.5) \cos \theta$$

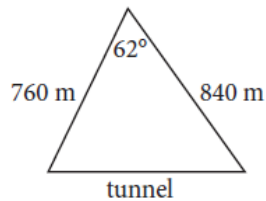
$$30.25 = 256 + 272.25 - 528 \cos \theta$$

$$-498 = -528 \cos \theta$$

$$0.94 = \cos \theta$$

$$19^\circ = \theta$$

Dahlwal is an engineer. For his latest contract, he has to determine the length of a tunnel that is to be built through a mountain. He chooses a point facing the mountain. He measures a distance of 840 m from one end of the tunnel to the point and a distance of 760 m from the other end of the tunnel to the point. The angle at the point to both ends of the tunnel is  $62^\circ$ . Calculate the length of the proposed tunnel to the nearest metre.



Decide which method to use.

$$t^2 = 760^2 + 840^2 - 2(760)(840)\cos 62$$

$$t^2 = 683778.71$$

$$t = 826.91$$

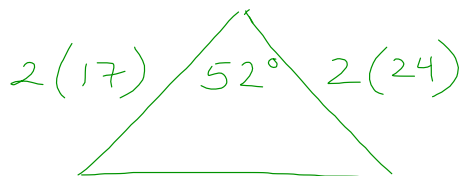
$\therefore$  The length of the tunnel is 826.9 m.

Two roads diverge at a  $52^\circ$  angle. Two bike riders take separate routes at 17 km/h and 24 km/h. How far apart are they after 2 hours?

Draw a triangle.

Label.

Decide which method to use.



$$x^2 = 34^2 + 48^2 - 2(34)(48)\cos 52^\circ$$

$$= 1450.48$$

$$x = 38.09$$

$\therefore$  They are 38 kms apart.

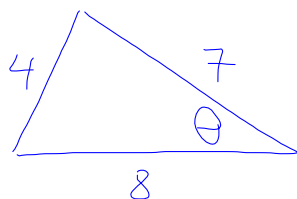
### 3C - 1 - day 5 - Cosine Law.notebook

Find the measure of the smallest angle (across from the shortest side) in a triangle with sides 4 m, 7m, and 8 m.

Draw a triangle.

Label.

Decide which method to use.



$$4^2 = 8^2 + 7^2 - 2(8)(7) \cos \theta$$

$$16 = 64 + 49 - 112 \times \cos \theta$$

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$$-97 = -112 \cos \theta$$

$$0.866 = \cos \theta$$

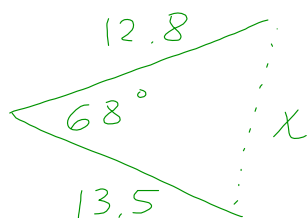
$$30^\circ = \theta \quad \leftarrow \cos^{-1}$$

A flock of Canada geese are flying in a V-formation that forms an angle of  $68^\circ$ . The lead goose is 12.8 m from the last goose on the left and 13.5 m from the last goose on the right. How far apart are the last two geese in the V-formation? Round your answer to the nearest tenth of a metre.

Draw a triangle.

Label.

Decide which method to use.



$$x^2 = 12.8^2 + 13.5^2 - 2(12.8)(13.5) \cos 68^\circ$$

$$= 216.6$$

$$x = 14.7 \quad \therefore \text{They are } 15 \text{ m apart.}$$

Have a great day!