EXPONENTIAL GROWTH

Learning Goals
- **formula for exponential growth and decay**
- **use the formula in real life examples**

Handou

5.2.1 Investigating Exponential Growth: Pizza Toppings

INTRODUCTION In this activity, you and your partner will investigate the number of different pizzas that can be created for a given number of available toppings.

INSTRUCTIONS

1. Fill in the chart below by drawing all the different pizzas that can be created by choosing all, some or none of the available toppings indicated.

Toppings Available	Different Pizza Drawings	
none	plain	(1)
Cheese	plain , cheese	(2)
Cheese, Pepperoni	plain , c , p , c + p	(4)
Cheese, Pepperoni, Mushrooms	pl, c, p, m, mc, mp, cp, cpm	(8)
Cheese, Pepperoni, Mushrooms, Bacon		(16)

2. a) Use your information from question 1 to complete the table below

Number of Available Toppings	Number of Different Pizzas	First Differences	Ratio
0	$1 = 2^0$		
1	$2 = 2^1$	1	
2	$4 = 2^2$	2	$\frac{2}{1} = 2$
3	$8 = 2^3$	$8-4=4$	$\frac{4}{2} = 2$
4	$16 = 2^4$	$16-8=8$	$\frac{8}{4} = 2$

same

$y = 2^x$

$\therefore$ exponential

b) What do the First Differences tell us about this data? Why?

exponential ratios are the same

c) Is there a constant ratio between consecutive values in the column titled Number of Different Pizzas?

2

How does this value relate to the pattern in the First Differences?

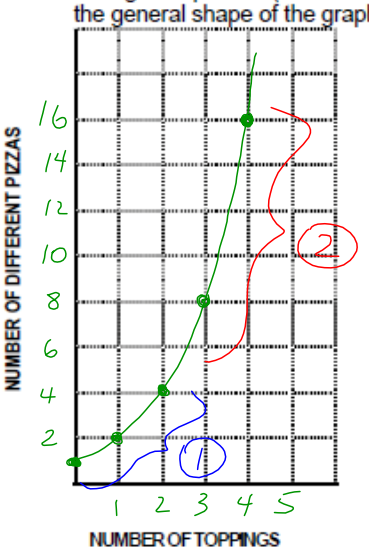
$\times 2$

c) Predict how many different pizzas could be created from 5 toppings. Justify your answer.

$16 \times 2 = 32$

5.2.1 Investigating Exponential Growth: Pizza Toppings (continued)

3. Neatly sketch a graph of your results from question 2 on the grid below. Draw a smooth curve through the points. (Note: This is DISCRETE data; however, the smooth curve assists in seeing the general shape of the graph.)



4. Using the graph comment on the shape of the curve. Use words such as the following in your description: increasing, decreasing, quickly, slowly.

① increasing slowly

② increasing fast

5. **Complete this statement**: As the number of toppings increases by 1, the number of different pizza combinations ___doubles___

6. Predict how many different pizzas can be created if there are nine available toppings. Clearly explain how you made your prediction.

$y = 2^x$

$y = 2^9 = 512$

7. If a restaurant owner would like to offer 200 different pizza combinations, what is the minimum number of available toppings she would need? Explain your reasoning.

$2^8 = 256$ $\quad 2^{7.65} = 200.8$

$2^7 = 128$ $\quad \therefore$ 8 toppings

8. a) Your local pizza parlour offers you the choice of 15 different toppings. If you were to eat a different pizza every day, how many years would it take for you to try every possible one? (Hint: There are 365 days in a year.)

$2^{15} = 32768$ pizzas $\quad \frac{32768}{365} = 89.7$ years

b) Does this answer surprise you? Why or why not?

At the end the graph will increase very fast.

Exponential Growth

$$P_n = P_o(1+r)^n$$

- n: Number of Growth Periods
- P_n: Final Amount
- P_o: Initial Amount
- r: Rate of Growth (as a decimal)

Joe deposits $1000.00 into a savings account that pays 3% interest per year.
How much money will he have at the end of 5 years?

$$P_n = P_0(1+r)^n$$

$$= 1000(1+0.03)^5$$

$$= 1159.27$$

∴ he will have $1159.27

In 1950 the world's population was 2.5 billion, with a growth rate of 1.7%/year.
Determine what year the population will reach 7.5 billion.

$$P_n = P_0(1+r)^n$$

$$7.5 = 2.5(1+0.017)^n$$

(x, (), Exp, n, ?)

$$\frac{7.5}{2.5} = \frac{2.5(1.017)^n}{2.5}$$

$$3 = 1.017^n$$

Guess and Check

$1.017^{15} = 1.27$

$1.017^{65} = 2.99$

$1.017^{66} = 3.04$

$1.017^{65.2} = 3.001$

∴ 65 years (year 2015)

On the Boards...

1. Cells in a culture are growing by a factor of 3.45 per day. The number of cells in the culture, N, can be estimated using the formula $N = 1000(3.45)^d$, where d is the number of days.

a) Use technology to plot a graph of this relation.

b) How many cells does this culture begin with?

c) How many cells would there be after 1 day?

d) How many cells would there be after 5 days?

b.) 1000

c.) $1000(3.45)^1 = 3450$

d.) $1000(3.45)^5 = 488759.8$

The population of Windsor, Ontario was 200000 in 1995. The city has an average growth rate of 12%. How many people will live in Windsor in the year 2025?

$P_n = P_0(1+r)^n$ $\quad n = 30$

$= 200000(1+0.012)^{30}$

$= 286052.25$

$\therefore$ 286052 people

Seatwork

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