EXPONENTIAL DECAY

Learning Goals
- **investigate exponential decay**
- **learn formulas for decay**
- **use formulas in real life situations**

5.3.3 Investigating Exponential Decay: Car Depreciation

INTRODUCTION: Depreciation is the decline in a car's value over the course of its useful life. It's something new-car buyers dread. Most modern domestic vehicles typically depreciate at a rate of 15%-20% per year depending on the model of the car.

INSTRUCTIONS

1. A 2007 Ford Mustang GT convertible is valued at $32 000 and depreciates on average at 20% per year.

Since the car is depreciating at 20% per year, the remaining value at the end of the first year is 80% of the original value.

Therefore, to find the depreciated value, multiply the previous year's value by 0.8

2. Complete the following table to calculate the value of the car at the end of each of the first five years of ownership.

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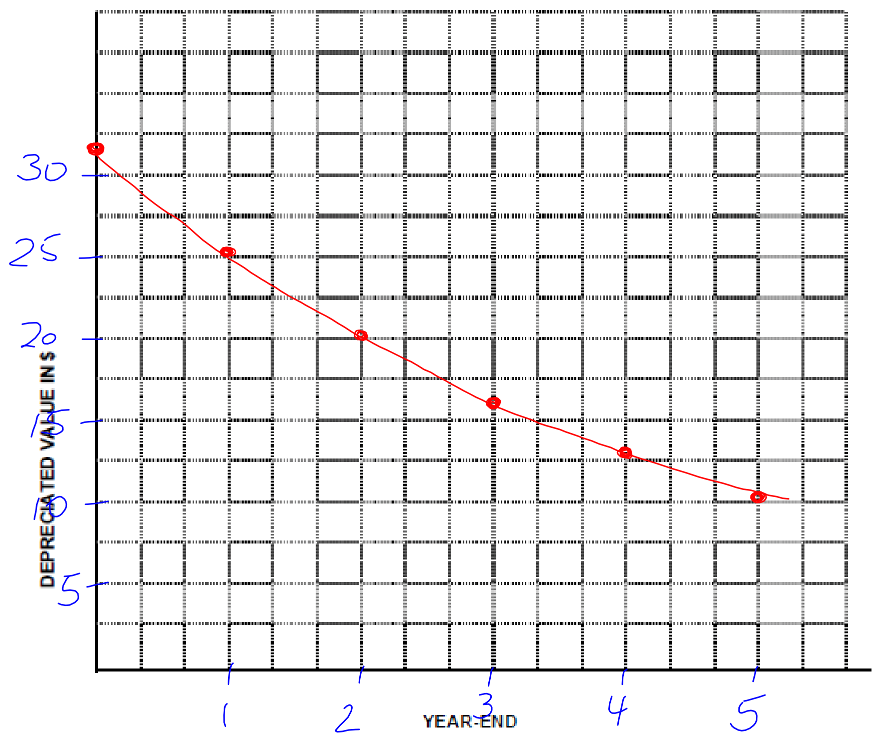
Year-end	Depreciated Value in $	Show Calculation in this column
0	32 000	
1	25 600	32000 (0.8)
2	20480	25600 (0.8)
3	16384	
4	13107.20	
5	10485.80	

3. What would you expect the constant ratio to be for this example. Justify your answer.

0.8

5.3.3 Investigating Exponential Decay: Car Depreciation (continued)

4. Neatly sketch a graph of your data. Draw a smooth curve through the points.



5. If the depreciation was 15% per year, how would the constant ratio change?

100% − 15% = 85%

∴ ratio is 0.85

∴ multiply each value by 0.85

∴ curve will be <u>less</u> steep

Decay

$$P_{(n)} = P_o(1 - r)^n$$

final amount

initial amount

going down

rate of decay (in decimals)

time

Half Life

- the time it takes for half of the material to decay
- often used with radioactive elements such as Carbon-14, Plutonium, Uranium
- often used with drugs or any other substances that the body metabolizes

  ex. caffeine and alcohol

Half Life

time

$$P(t) = P_o\,(1/2)^{t/h}$$

half-life

final amount

initial amount

The number of deer on an island is 700. Researchers estimate a 5% decline in the population every year. How many deer will there be after 10 years?

$$P_n = P_o(1-r)^n$$

$$= 700(1-0.05)^{10}$$

$$= 419.116$$

$\therefore$ 419 deer left

Archeologists use C-14 to estimate the age of artifacts. C-14 has a half-life of 5730 years. A newly found bone contained 20% of the initial carbon. Determine the age of the bone.

$$P(t) = P_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$0.20 = 1\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

final amount

start at 100%

$$0.20 = (0.5)^{\frac{t}{5730}}$$

To find an exponent
$\Rightarrow$ Guess and Check

$$0.5^{\frac{200}{5730}} = 0.97$$

$$0.5^{\frac{700}{5730}} = 0.91$$

$$0.5^{\frac{6000}{5730}} = 0.48$$

$$0.5^{\frac{12500}{5730}} = 0.22$$

$$0.5^{\frac{13250}{5730}} = 0.201$$

$\therefore$ bone is $\sim$ 13250 yrs old

On the Boards...

Certain types of minor skin wounds heal at a rate modelled by the relation $W = W_0\left(\frac{1}{2}\right)^{0.36t}$, where W is the area of the wound currently, in square millimetres, W_0 is the initial wound area, and t is the time, in days, after the wound has been dressed. What will be the area of a 25 mm^2 wound after

a) 1 day?

$= 25\left(\frac{1}{2}\right)^{0.36(1)}$

$= 19.48$

b) 4 days?

$= 25(0.5)^{0.36(4)}$

$= 9.21$

The relation $T = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$ can be used to determine the length of time, t, in hours, that milk of a certain fat content will remain fresh. T is the storage temperature, in degrees Celsius.

a) What is the freshness half-life of milk? 10h

~~**b)** Graph the relation.~~

c) How long will milk keep fresh at 22°C? at 4°C?

$22 = 190(0.5)^{\frac{t}{10}}$

$4 = 190(0.5)^{\frac{t}{10}}$

Guess and Check

$t = 31.1$

$t = 55.7$

The remaining concentration of a particular drug in a person's bloodstream is modelled by the relation $C = C_0\left(\frac{1}{2}\right)^{\frac{t}{4}}$, where C is the remaining concentration of drug in the bloodstream in milligrams per millilitre of blood, C_0 is the initial concentration, and t is the time, in hours, that the drug is in the bloodstream.

a) What is the half-life of this drug?

b) A nurse gave a patient this drug. The concentration was 40 mg/mL, at 10:15 A.M. What will the concentration at

i) 3:15 P.M.? **ii)** 10:00 P.M.?

5 hours

$40(0.5)^{\frac{5}{4}}$

$= 16.8$

11 h 45 min

11.75 h

$40(0.5)^{\frac{11.75}{4}}$

$= 5.22$

Seatwork

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